Conservation of Energy & Vertical Circles

2005 SP

10. A hollow circular cylinder of radius 0·4 metres is fixed with its axis horizontal. A body of mass 0·5 kg moves on the inside smooth surface of the cylinder and in a vertical plane perpendicular to the cylinder's axis. Find the least speed which the body must have at the lowest point of its path if it travels in complete circles. **6**

If the body was projected from the lowest point, A, inside the cylinder, with initial speed $\sqrt{\frac{3g}{2}}$ m s^{-1}, where g is the magnitude of the acceleration due to gravity, to travel in the same vertical plane as before, show that it would leave the cylinder at a point such that the radius of the circle to that point makes an angle of $\cos^{-1}\frac{7}{12}$ with the upward vertical. **5**

2006

A8. A mass m kilograms is attached to one end, A, of a light inextensible string of length L metres, the other end of which is fixed at a point O. Initially the mass hangs vertically below O with the string taut. The mass is then given a horizontal speed of $\sqrt{\frac{7}{2}gL}$ ms^{-1}, causing it to start to travel in a vertical circle of centre O. Subsequently, the string OA makes an angle θ with the downward vertical through O.

(*a*) When $\theta = 45°$, find expressions for:

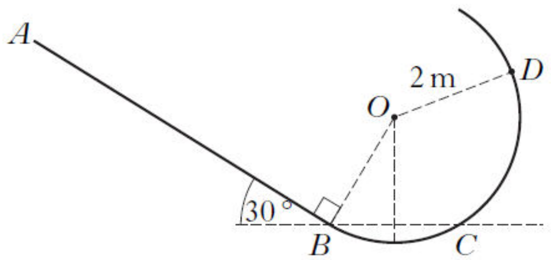
(i) the speed of the mass in terms of L and g; **4**

(ii) the magnitude of the tension in the string, in terms of m and g. **3**

(*b*) Determine the value of θ at which the string first becomes slack. **4**

2009

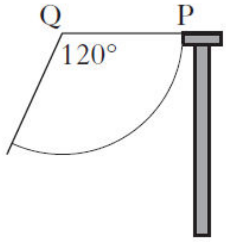
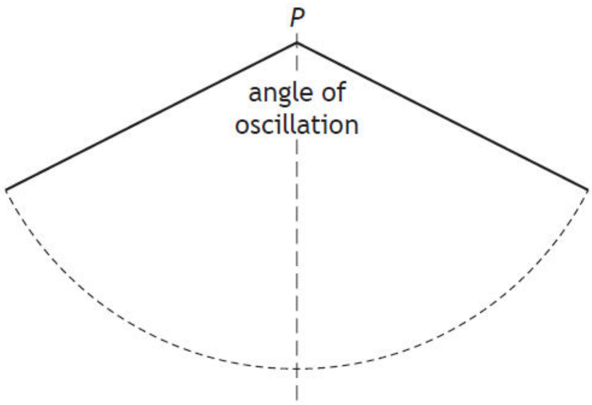
A10. A track consists of a rough, straight section AB which is inclined at an angle of $30°$ to the horizontal. The section AB is tangential to a semi-circular section of smooth track lying in the same vertical plane as AB. The centre of the semi-circle is at O and the radius is 2 m. The point C is at the same horizontal level as B.



A miniature sledge of mass 0·2 kg is released from rest at A. The section AB is 5 m long and provides a constant resistive force of 0·08 N to the motion of the sledge. The sledge continues on the smooth curved section, losing contact with the track at a point D. Calculate:

(*a*) the kinetic energy of the sledge at C; **4**

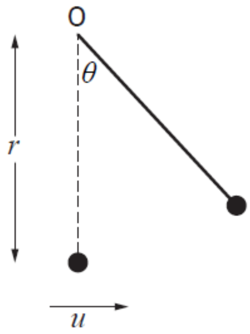
(*b*) the angle between OD and the horizontal. **6**

2013	**A8.** A smooth solid hemisphere of radius a metres is fixed with its plane face on a horizontal table and its curved surface uppermost. A particle P of mass m kilograms is placed at the highest point on the hemisphere and given an initial horizontal speed $\sqrt{\frac{ag}{2}}\ \mathrm{m\,s^{-1}}$. The particle moves along the curved surface of the hemisphere until it leaves the surface at Q. Calculate the angle between the tangent at Q and the horizontal, and find an expression for the speed of the particle at Q. **6**
2015	**A6.** An acrobat of mass 60 kilograms starts her routine sitting on a platform P. She is holding a rope of length 4 metres which is attached to a fixed support Q on the same horizontal level as P. With the rope taut and horizontal, she drops off the platform and swings in a circular arc. When she has swung through 120° she lets go of the rope.  (*a*) Show that her speed on the point of release is approximately $8 \cdot 24\ \mathrm{m\,s^{-1}}$ and find the tension in the rope at that time **4** After letting go of the rope, she moves freely under gravity. (*b*) Find how far she rises before starting her descent. **2**
2014 & 2016 EX	**14.** A light rod PQ of length $0 \cdot 9\ \mathrm{m}$ has a particle of mass $3\ \mathrm{kg}$ attached at Q. The rod is free to rotate in a vertical plane about P. When Q is vertically below P the mass is given a horizontal velocity $u\ \mathrm{ms^{-1}}$ causing the rod to move in the vertical plane. (a) Show that, for the rod to complete a full circle, $u > \sqrt{\frac{18g}{5}}$. **3** On another occasion, the horizontal velocity given to the mass is $4\ \mathrm{ms^{-1}}$ and the rod oscillates.  (b) Find the angle of oscillation and the greatest tension in the rod during its motion. **7**

2016

17. A light inextensible string of length r metres has one end attached to a fixed point O and the other end is attached to a particle of mass m kilograms.

From its equilibrium position, the particle is given a horizontal velocity $u\,\mathrm{m\,s^{-1}}$, as shown in the diagram.



(a) (i) Show that the tension, T, in the string can be expressed as

$$T = \frac{mu^2}{r} + mg\left(3\cos\theta - 2\right)$$

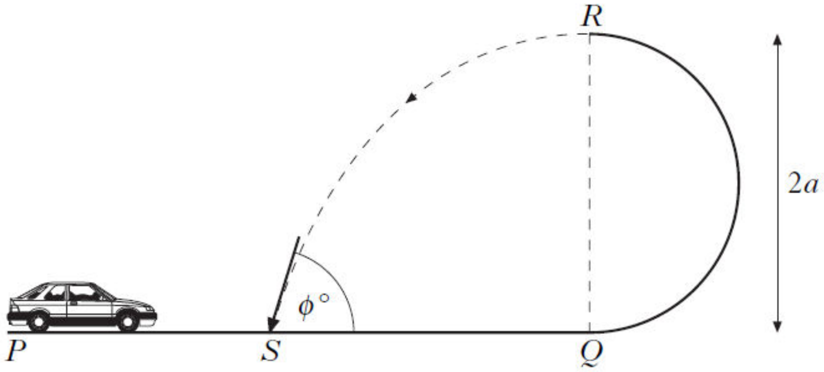
where θ is the angle between the string and the downward vertical through O. **4**

(ii) Determine a condition for u in terms of r and g, so that the particle executes a complete circle. **2**

(b) Given that the value of u is $2\sqrt{rg}$, find an expression in terms of r for the height of the particle above its starting position when the string goes slack. **3**

2007

A9. The diagram below shows a smooth plastic track. The section PQ is horizontal and the section QR is semi-circular and in the same plane as PQ. The diameter QR is vertical and has length $2a$ metres.



A toy car is projected along PQ with speed $3\sqrt{ga}\ \mathrm{m\,s^{-1}}$. The car travels around the track to R, where it leaves the track horizontally, landing on PQ at the point S, where the angle between the car's trajectory and the line SQ is $\phi°$.

(*a*) Find the speed of the car at R, expressing your answer in the form $\sqrt{kga}$, where k is a constant. **3**

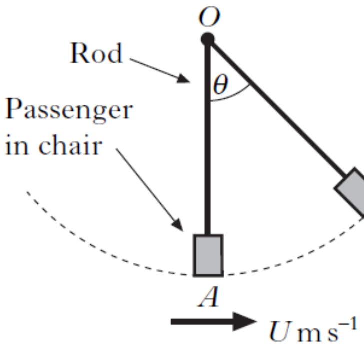
(*b*) Show that at R the car is in contact with the track. **2**

(*c*) Show that $SQ = 2\sqrt{5}a$ metres. **3**

(*d*) Find the exact value of $\tan\phi°$. **2**

2010

A11. A fairground ride consists of a rod that is free to rotate in the vertical plane about a fixed point O. A passenger sits in a chair that is attached to the other end of the rod, as shown in the diagram.



The ride can be modelled as a particle attached to a light inextensible rod. The angle between the rod and the vertical OA is θ radians, the length of the rod is R metres and the mass of the particle is M kg. The maximum value of the angular displacement θ is denoted by α. The particle is initially at a point A, vertically below the fixed point O.

(*a*) The particle is given an initial speed of $U\,\mathrm{m\,s^{-1}}$ when it is at A and this causes the particle to oscillate through an arc with $\alpha = \frac{\pi}{4}$.

Show that

$$U^2 = gR(2 - \sqrt{2}).$$ **2**

The amplitude of the oscillations is increased such that $-\pi < \theta < \pi$ and the speed of the particle at A is $u\,\mathrm{m\,s^{-1}}$.

(*b*) Find an expression for the speed $v\,\mathrm{m\,s^{-1}}$ of the particle at any point in the oscillation, in terms of u, g, R and θ. **2**

(*c*) Given that the tension in the rod, as a function of angle θ, is denoted by $T(\theta)$ and assuming that $\frac{3\pi}{4} < \alpha < \pi$, show that

$$T\left(\frac{\pi}{4}\right) - T\left(\frac{3\pi}{4}\right) = kMg$$

where k is a constant to be obtained. **6**

2008

A9. A ball of mass 0·1 kg is released from a point A at the top of a smooth runway AO. The point O is 1 metre above ground level and, when the ball reaches O, it falls to the ground under the action of gravity.

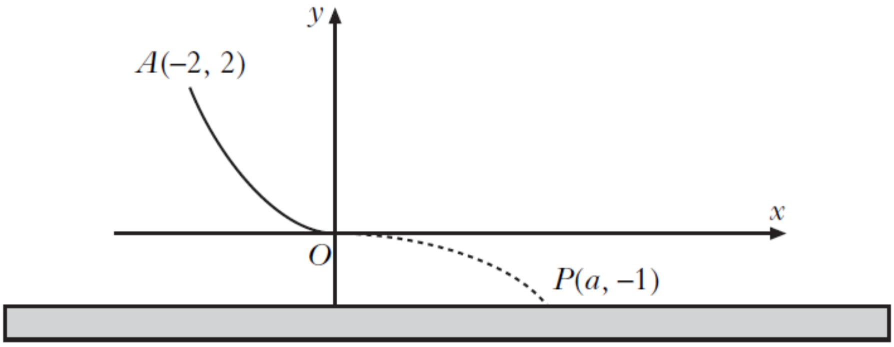


Figure 1

(*a*) Relative to the axes shown in Figure 1, the runway is modelled by the curve $y = \frac{1}{2}x^2$. The point A is $(-2,\ 2)$ and O is the origin. The ball reaches the ground at $P(a, -1)$. Calculate the value of a. **5**

(*b*) The track is modified to run between the same two points A and O with the shape modelled by the equation $y = x(x + 1)$ as shown in Figure 2.

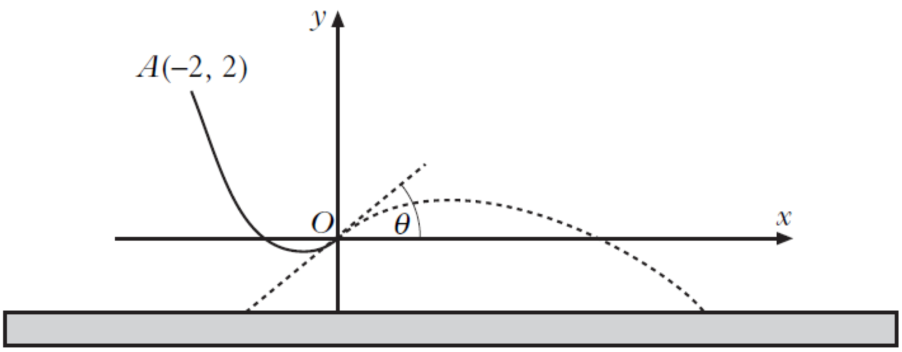


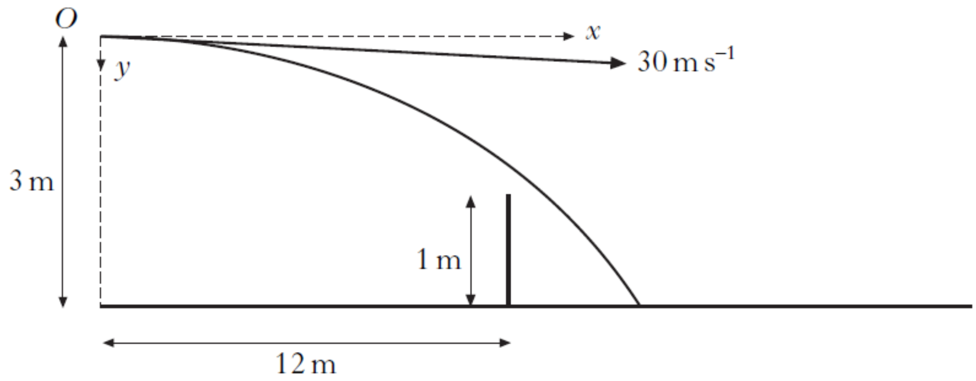
Figure 2

Calculate the maximum height above ground level attained by the ball after it has passed through O. **6**

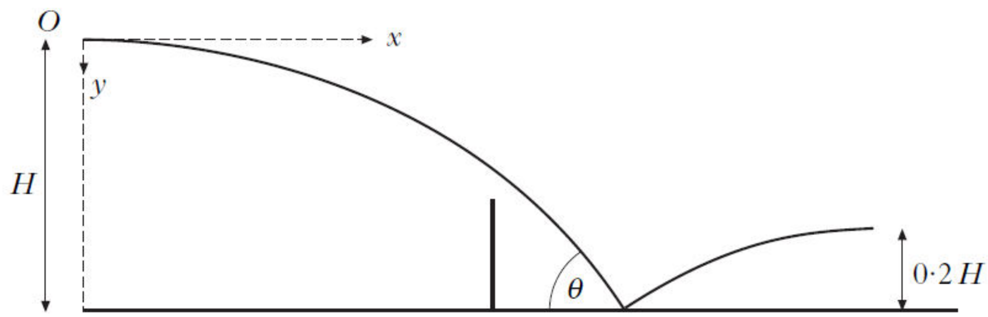
2011

A9. (*a*) A tennis ball is projected with speed $30\,\mathrm{m\,s^{-1}}$ at angle of $5°$ below the horizontal, from a height of 3 metres and horizontal distance of 12 metres from the net which has height one metre.

By how much does the ball clear the net? **3**



(*b*) A second ball is projected from height H metres with initial velocity vector $\mathbf{u} = \sqrt{6gH}\,\mathbf{i}$, where $\mathbf{i}$ is a unit vector along the Ox direction. At the instant the ball hits the tennis court, the velocity vector makes an acute angle θ with the surface of the court.



Obtain an expression for the speed of impact of the ball with the court in terms of g and H, and determine angle θ. **4**

During the bounce, the ball loses 50% of its kinetic energy and continues, reaching a maximum height of $0{\cdot}2\,H$ metres.

Obtain an expression, in terms of g and H, for the speed of the ball when it reaches its maximum height. **3**